Solar Dynamics, Rotation, Convection and Overshoot

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ABSTRACT

We discuss recent observational, theoretical and modeling progress made in understanding the Sun's internal dynamics, including its rotation, meridional flow, convection and overshoot. Over the past few decades, substantial theoretical and observational effort has gone into appreciating these aspects of solar dynamics. A review of these observations, related helioseismic methodology and inference and computational results in relation to these problems is undertaken here.

1. Introduction

Rotation and convection play an important role in solar and stellar evolution. They also play a crucial role in generating the magnetic activity exhibited by the Sun and other stars.

Rotationally induced instabilities, convection and overshoot mix chemical elements within stars, with possible consequences both for the observed chemical abundances at the surface of the star and for the availability of fuel for the nuclear energy generation in the star's core. Overshoot too can affect the chemical balance of the star, in the case of the Sun for example transporting a fraction of the fragile elements lithium and beryllium to regions where the temperature is sufficiently high for these elements to be destroyed.

Rotation and convection, and their interplay, can create dynamo action and generate magnetic field in a star. In the Sun, the overshoot layer at the base of the solar convective envelope may also play a crucial role in storing the magnetic field until it is strong enough to become buoyantly unstable and rise through the convection zone to the surface. The convection interacts with the rising magnetic field and largely determines how the magnetic field appears at the surface.

Observational data on the rotation, convection and other internal dynamics of the Sun were formerly restricted to what could be observed at the surface: granulation and supergranulation set up by the convective motions, and the surface rotation rate inferred from spectroscopic measurements and from observing the motion of tracers of the rotation such as sunspots and other surface features. This situation has been revolutionized by helioseismology, the observation and analysis

of global oscillations and more localized acoustic wave propagation. At the same time as helio-seismology has revolutionized the observational study of the Sun's internal dynamics, advances in numerical simulations – greatly assisted by the massive increase in computational power of modern supercomputers – have enabled great advances in the modeling and consequent understanding of the internal dynamics of the Sun and other stars. This chapter will discuss these observational and theoretical advances.

2. Flows in the solar interior

The global-scale flows in the Sun are its rotation and meridional circulation. Flows on smaller scales include convection, from granular scales to the putative giant-cell convection, and outflows and inflows around and beneath active regions. This section gives a brief overview of these different flows. The succeeding sections will then focus on what has been learned observationally about these flows from helioseismology, and the theoretical advances being made in modeling them.

2.1. Rotation

Like other stars, the Sun acquired its angular momentum from the interstellar gas cloud from which it formed. As the gas contracts to form a star, unless there is some extremely efficient mechanism for losing angular momentum, the proto-star spins up so as to rotate much faster than the parent cloud. Thus young stars are typically observed to be fast rotators, and that was presumably the case for the Sun also. Over the next 4-5 billion years, the Sun would have lost angular momentum from its surface layers via the solar wind (see, e.g., Bouvier 2013; Gallet and Bouvier 2013, for models). If there were no angular-momentum transport mechanisms at play in the solar interior, this would result in a fast-rotating core and a slowly rotating envelope. The evidence from helioseismology (Section 3) does not support such a picture, indicating that the radiative interior is rotating nearly uniformly at a rate intermediate between the polar and equatorial rate of the convection zone. This indicates that one or more mechanisms - magnetic fields, transport by rotationally induced instabilities and by gravitational waves (see review by, e.g. Mathis 2013) – have systematically extracted angular momentum from the radiative interior and redistributed the residual, suppressing rotational gradients. The solar surface has long been observed to rotate differentially, with the mid- and high-latitude regions rotating more slowly than the equatorial region. That this differential rotation largely persists with depth throughout the solar convective envelope is now established observationally by helioseismology. Angular momentum transport must maintain this differential rotation, and the mechanisms for that are addressed below in Section 4.1.

2.2. Meridional circulation

There is a poleward meridional circulation in the near-surface layers of the Sun in both the north and south hemispheres. Mass conservation dictates that their must be a return, equatorward, flow somewhere in the solar interior, and this is often presumed to occur near the base of the convection zone. However, whether a single meridional cell spans the whole depth of the convection zone or whether it is shallower, with perhaps other meridional cells stacked beneath the one nearest the surface, is a matter of active discussion. There is also uncertainty about whether the poleward flow extends right up to the poles, or whether there might be a counter cell at high latitudes. There are marginal observations at best to say whether or not there is any meridional circulation in the Sun's radiative interior, though there are theoretical arguments for the existence of a slow, Eddington-Sweet circulation (e.g. Zahn 1992a).

Determining the meridional flow as a function of radius and latitude is of considerable interest. This is to a large extent driven by the importance of the meridional flow in certain dynamo theories.

2.3. Convection and convective overshoot

Energy transport in the optically thin solar photospheric layers transitions from being effected by convection to free-streaming radiation. A spatio-temporal power spectrum of photospheric flows reveals granular and supergranular scales. Observed properties of granules, such as spatial scales, radiative intensity and spectral-line formation are highly accurately reproduced by numerical simulations (e.g., Stein and Nordlund 2000; Vögler et al. 2005; Nordlund et al. 2009). One may conjecture that the success of the simulations in spite of being in an entirely different parameter regime is due to the small scale height ($\sim$200 km; compare with radius of the Sun, $R_\odot \sim 700,000$ km), leading to the strong expansion of convective upflows, thereby smoothing out large fluctuations and making them almost laminar (Nordlund et al. 1997). The effects of turbulence are restricted to the downflows, where they appear to not have a big impact on the observable near-surface dynamics. Thus incorporating the ingredients of an accurate equation of state and background stratification lead to a high-fidelity reproduction of line formation and spatial scales.

Ostensibly, modeling convection in the solar interior presents a more formidable challenge, since there appear to be no dominant physical ingredients that are concurrently computationally tractable. Interior convection is likely governed by aspects more difficult to model, such as the integrity of descending plumes to diffusion and various instabilities (Rast 1998). Further, solar convection is governed by extreme parameters (Prandtl number $\sim 10^{-6} - 10^{-4}$, Rayleigh number $\sim 10^{19} - 10^{24}$, and Reynolds number $\sim 10^{12} - 10^{16}$; Miesch 2005), which makes fully resolved three-dimensional direct numerical simulations impossible for the foreseeable future. It is likewise difficult to reproduce solar parameter regimes in laboratory experiments.

Phenomenology such as mixing-length theory (MLT) treats convective transport as being ef-

fected by parcels of fluid of specified spatial and velocity scales, coherent over a length scale termed the *mixing length*. Despite its overt simplicity (Weiss et al. 2004), it has been remarkably successful as an integral component of stellar structure models (e.g. di Mauro et al. 2010) and in describing the sub-photospheric stratification and heat transport in local solar convection simulations (Trampedach and Stein 2011). Because density and pressure scale heights increase with depth, MLT posits a corresponding increase in spatial convective scale (while velocities reduce), suggesting the existence of large convective cells, the so-called *giant cells*. Three-dimensional simulations of global convection have been performed at increasing resolutions over the past few years (Miesch et al. 2008; Ghizaru et al. 2010; Käpylä et al. 2010, 2011; Guerrero et al. 2013a; Hotta et al. 2014a,b), most invoking the *anelastic approximation* (Gough 1969), a regime describing low-Mach number strongly non-Boussinesq stratified convection. These simulations predict large-scale convective turnover and velocity amplitudes comparable to those of MLT. Considerable effort has been spent in attempting surface (Hathaway et al. 2000, 2013) and interior detection (Duvall et al. 1993; Duvall 2003) of giant cells but unambiguous identification remain elusive. This difficulty in detection suggests that the amplitudes of giant cells may be significantly less than predicted by MLT and convection simulations, potentially posing serious challenges to our understanding of deep solar convection (section 3.3).

At the base of the convection zone, convective motions are quickly decelerated by the steep subadiabatic stratification of the radiative interior. This thin overshoot region (section 3.3) coincides with the rotational shear layer known as the solar tachocline (section 3.1) as first posited by Spiegel and Zahn (1992), possibly suggesting a causal connection. The dynamics in this region are complex, involving penetrative convection and internal waves interacting with a stably-stratified, magnetized shear flow and the much longer dynamical time scales of the radiative zone (Hughes et al. 2007).

3. Helioseismic constraints on the Sun's internal dynamics

One of the great successes of helioseismology has been to measure the rotation of much of the solar interior, excluding the solar core and a region around the Sun's rotation axis. Much of this has been achieved using global-mode helioseismology (e.g., Deubner and Gough 1984), which proceeds by the analysis of the observed properties of global resonant modes of oscillation of the Sun, in particular the mode frequencies. See for example the excellent review by Howe (2009). Global-mode helioseismology has been complemented by the development and application of a number of other approaches known collectively as local helioseismology. These include time-distance helioseismology (Duvall et al. 1993), acoustic holography (Lindsey and Braun 1997), and ring-diagram analysis (Hill 1988). Local helioseismology has been particularly used to probe the Sun's meridional (i.e. north-south) circulation and convective motions in the upper part of the Sun's convection zone, and motions around and under active regions (e.g., Komm et al. 2005).

3.1. Rotation

The global-mode frequencies ω_{nlm} of the Sun are labeled by the radial order n, and the degree l and azimuthal order m of the spherical harmonic that describes the horizontal structure of the mode. In the absence of rotation and other departures from perfect spherical symmetry, the frequencies would be independent of m. The Sun's rotation lifts that degeneracy. The difference in frequency between modes with like values of n and l but different values of m is called the splitting. For a star with internal rotation rate $\Omega(r, \theta)$ (with respect to spherical polar coordinates r, θ, ϕ), the rotational splitting is given by (Hansen et al. 1977)

$$\delta\omega_{nlm} \equiv \omega_{nlm} - \omega_{nl0} = -i \int \rho \left(\xi_{nlm}^{*} \cdot \frac{\partial}{\partial \phi} \xi_{nlm} \right) \Omega \, dV \bigg/ \int \rho \xi_{nlm}^{*} \cdot \xi_{nlm} dV \tag{1}$$

where $\xi_{n;m}$ is the radial displacement eigenvector of the mode, ρ is the density, and the integrals are over the interior volume of the star. This can be conveniently rewritten as

$$\delta\omega_{nlm} = m \int_{0}^{R} \int_{0}^{\pi} K_{nlm}(r, \theta) \Omega(r, \theta) r d\theta dr \tag{2}$$

where the K_{nlm} are known as rotational splitting kernels. If the static structure of the stellar interior is known, the kernels can be calculated, so that the only unknown in equation (2) is the rotation rate $\Omega(r, \theta)$. Measurements of the Sun's mode frequencies therefore provide a set of observational constraints of the form (2), which may be used to infer (or at least constrain) the solar internal rotation rate.

The solar internal rotation rate is thus fairly well determined from the photosphere down to beneath the convection zone, excluding a region around the poles. See e.g. Thompson et al. (1996), Schou et al. (1998), Howe et al. (2011), Eff-Darwich and Korzennik (2013) for examples of results using data from, respectively, GONG (Harvey et al. 1996), MDI (Scherrer et al. 1995) and HMI (Schou et al. 2012).

A typical rotation profile determined from helioseismology is shown in Fig. 1. The latitudinal variation of rotation observed at the surface largely persists through the convection zone, while in the radiative interior the results are consistent with solid-body rotation. But there are two rotational shear layers, a near-surface shear layer and another shear layer termed the tachocline that is located near the base of the convection zone.

Superimposed on the mean rotation profile at the surface and in the convection zone are weak but apparently coherent bands of faster and slower zonal flow that migrate in latitude over the course of the solar cycle (Fig. 2). These have been dubbed "torsional oscillations" (Howard and Labonte 1980; Schou 1999; Howe et al. 2000; Vorontsov et al. 2002). The causal connection between the flows and the magnetic manifestations of the solar cycle are unclear.
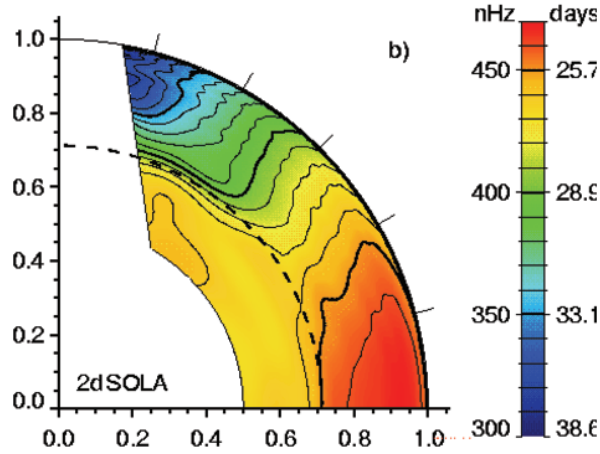
Fig. 1.— Solar internal rotation as inferred from MDI observations. Contours of isorotation are shown. (Adapted from Schou et al. 1998)

3.2.   Meridional circulation

Unlike the rotation, the meridional flow does not perturb the mode frequencies to first order. As a consequence most measurements of the meridional flow have been made using local helioseismology techniques, such as time-distance and ring-diagram analysis.

In principle, the time-distance measurements are quite straightforward. The N-S and S-N travel times are measured for a selection of latitude pairs and the results inverted to obtain the meridional flow as a function of latitude, depth and time. In reality, and as illustrated in Zhao et al. (2012), measurements of wave travel times (and seismic measurements in general) are prone to systematic errors, especially when they are taken at spatial locations away from the disk center. One manifestation of these errors is that the derived flows in the interior depend on the type of observations used, an unsatisfactory state of affairs. Making the assumption that the error is due to a center-to-limb time shift, Zhao et al. were able to diminish the discrepancy between observables and obtain an estimate of the meridional circulation over much of the solar convection zone. Remarkably, Zhao et al. (2013) found that the meridional circulation pattern consists of two circulation cells in the depth direction (Figure 3), which if it were to hold up, will challenge some solar dynamo models (e.g. Jouve and Brun 2007). While suggestions have been made regarding the origin of the systematic errors (e.g., Baldner and Schou 2012), no definite conclusion has been reached so far.

Ring-diagram analysis has also produced a number of interesting results on the meridional circulation (e.g., González Hernández et al. 1999, 2006, 2008, 2010; Haber et al. 2006). The inferred meridional circulation exhibits temporal variations, including the occasional appearance of an equatorward cell at high latitudes. Part of those variations is thought to be the result of a systematic error caused by uncertainty in the Carrington elements: the inferred equatorward meridional flow
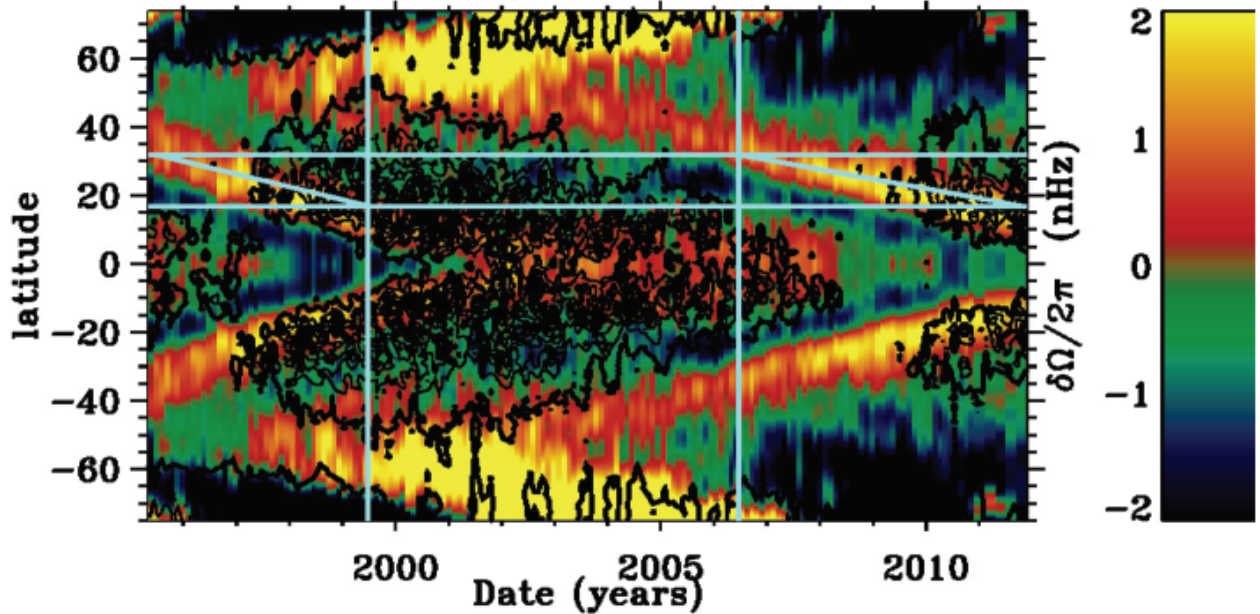
Fig. 2.— Torsional oscillations in the form of faster (red and yellow colors) and slower (green and blue/black colors) zonal flows relative to the long-term average rotation, as inferred from helioseismology in the near subsurface layers of the Sun using MDI and GONG observations. Contours of photospheric magnetic field strength are superimposed to indicate the location of contemporaneous surface activity. The left vertical blue line marks the date 1997.3, when low-latitude flows most closely matched the inference from the 2009.2 data. The vertical blue line on the right signifies 2006.4, most closely resembling the analysis from the earliest data (1996.5). The horizontal lines mark locations of the flow bands, while the slanted lines indicate equatorward migration. (After Howe et al. 2009, courtesy of Rachel Howe.)

at high latitutdes shows an annual variation wiht the B_0 angle (Zaatri et al. 2006). Nevertheless, a multi-cell structure in latitude is also seen in the Mount Wilson surface Doppler measurements (Ulrich 2010).

Lavely and Ritzwoller (1992) described a perturbation theoretical approach that included the study of general bulk flows. Following this theoretical framework, Roth and Stix (1999, 2008) studied the effect of giant cells and the meridional flow on the global mode frequencies. The effects are small, however, making it difficult for them to be measured and to be evaluated for helioseismic mappings of the solar interior. Recently, new approaches concentrate on investigating the perturbation of the eigenfunctions rather than the eigenfrequencies when studying the effect of the meridional flow on solar oscillations (Woodard 2000; Schad et al. 2011, 2012, 2013; Woodard et al. 2013). Since the meridional flow perturbs the solar model, the new oscillatory eigenstates expressed in the basis of the unperturbed eigenstates are mixtures of modes, which is often called mode coupling. Given an azimuthally symmetric meridional flow, such couplings only occur between

modes of identical azimuthal orders m. Depending on the complexity of the flow when expanded in terms of Legendre polynomials as a function of latitude, mode coupling may occur between modes that differ in l by the harmonic degree of the flow. As a consequence, power of one mode is expected to leak into another mode and vice versa. The cross-spectrum allows measuring this coupling of the modes. As systematic leakage is a dominant error source in the cross-spectra due to observational and instrumental constraints, detailed knowledge of the instrument and observation conditions must be taken into account.

Currently two approaches exist that estimate the meridional flow via cross-spectral analysis. The first one employs a fit of a model function to the observed cross-spectrum of modes that couple along the same ridge (Woodard et al. 2013). Figure 4 shows the resulting peak velocity of the meridional flow component with a harmonic degree of 2 as a function of the ratio $\nu/(l + 1/2)$, which is related to the inner turning point of the modes. Figure 4 would appear to indicate a rapid increase in the meridional flow below the surface. However, this may be an artifact related to the center-to-limb effect suggested by Zhao et al. (2012). The other approach evaluates ratios of the cross-spectral amplitudes (Schad et al. 2011, 2012) rather than a fit to the cross-spectra and includes inter-ridge couplings, too. Here the radial component of the meridional flow is measured and the horizontal meridional flow is obtained via mass conservation. Through the use of ratios, this method appears robust against systematic errors present in the cross-spectra. Figure 5 displays the result for the radial meridional flow measured at the equator and the horizontal meridional flow measured at 45° latitude as a function of fractional solar radius. Again, the harmonic degree of the flow component is 2.

Evaluating mode couplings caused from flow components with a harmonic degree up to 10, Schad et al. (2013) were able to measure some of these components down to a fractional solar radius of approximately 0.5. Only the flow components with harmonic degrees 2 and 8 deviate significantly from zero, and give a first hint on a meridional flow that is confined between the tachocline region and the solar surface and that exhibits a multi-cellular structure as a function of radius and latitude. Figure 6 shows a composite of the even meridional flow components.

With both techniques the inferred solar near-surface horizontal flow is consistent with sub-surface flow measurements from local helioseismology, with a poleward directed flow with a peak amplitude of order 20 m/s. However, we note that at greater depths the results shown in Figures 3 and those derived by Hathaway (2012) are inconsistent with Figure 6. Both Zhao et al. (2013) and Hathaway (2012) infer a return (equatorward) flow at $r \approx 0.9 R_\odot$ whereas Figure 6 suggests a flow that remains poleward up till a depth of $r = 0.8 R_\odot$. Further, although qualitatively similar up to a depth of $r = 0.9 R_\odot$ or so, the amplitudes of the return flow inferred by Hathaway (2012) and Zhao et al. (2013) are quantitatively different. Some of these discrepancies could be attributed to the fact that the analyzed data sets cover different time frames. Nevertheless, this is indicating that the results should be treated with some caution and that further work is needed to determine the source of the discrepancies.

3.3. Convection and overshoot

Using techniques of time-distance helioseismology (Duvall et al. 1993; Duvall 2003; Gizon et al. 2010), Hanasoge et al. (2012) placed stringent bounds on the interior convective velocity spectrum. Two-point correlations measured from finite temporal segments of length of the observed line-of-sight photospheric Doppler velocities, taken by HMI, were used in the analysis. These correlations were spatially averaged according to a deep-focusing geometry (Hanasoge et al. 2010) in order to image the interior. Convective coherence timescales (Spruit 1974; Gough 1977; Miesch et al. 2008) were taken into account in choosing the temporal length of the data. By construction, these measurements are sensitive to the 3 components of the underlying flowfield, i.e., longitudinal, latitudinal or radial, at specific depths of the solar interior ($r/R_\odot = 0.92, 0.96$). The constraints are shown in Figure 7. The stark difference between observations and simulations suggests that the convection in the Sun may be operating in a strongly non-MLT regime. A plume-based thermal transport mechanism in this alternative regime has been the subject of speculation by, e.g., Spruit (1997) and was explored further by Rempel (2005a). The alternate mechanism put forward by Spruit (1997) would be able to account for the outward thermal transport of a solar luminosity's worth of heat flux at extremely low flow speed. However, the transport of angular momentum, i.e., the maintenance of differential rotation and meridional circulation, is not as easily explained. Based on scaling arguments, Miesch et al. (2012) estimate the minimum convective kinetic energy (and associated Reynolds' stresses) required to sustain these large-scale flow circulations. Gizon and Birch (2012) provide an interesting comparison between seismology, simulation, and the phenomenology of Miesch et al. (2012).

Just as convection leads to an essentially adiabatically stratified envelope in the outer thirty per cent of the Sun, convective overshooting is expected to modify the stratification of the region in which it takes place (Skaley and Stix 1991; Deng and Xiong 2008). In the simplest picture, overshoot at the base of the convection zone leads to an extension of the adiabatically stratified region, with a more-or-less abrupt transition beneath that to the subadiabatic stratification of the radiative interior. Such a signature of a sharp transition in the sound-speed profile of the Sun has been sought with helioseismology (Basu et al. 1994; Monteiro et al. 1994; Roxburgh and Vorontsov 1994; Christensen-Dalsgaard et al. 1995) In principle, the amplitude of the abrupt transition can then be used to infer the extent of the overshoot region, and upper bounds have been quoted on the extent of the overshooting of about $0.005 R_\odot$ (Monteiro et al. 1994).

In fact, it appears that the transition between the convection zone and the radiative interior in the Sun is actually smoother even than in solar models that have no convective overshooting, which is a challenge to the simple picture above. A different model of convective overshoot has been proposed by Rempel (2004) wherein the overshoot is modeled with discrete plumes with a spectrum of strengths and hence depths of penetration into the radiative interior (also see Zahn 1991, for models of overshoot). Depending on the spectrum adopted, this can give a smoother transition, indeed the subadiabatic stratification can occur even towards the bottom of the convective region. Christensen-Dalsgaard et al. (2011) investigated the seismic signature of such models, compared

with helioseismic observations, and found that some models of this class fitted the observations better than the simpler models without (the "Standard solar model") or with overshoot. They concluded that overshoot is necessary to improve the agreement between models and helioseismic constraints, that the required stratification profiles are outside the realm of classic "ballistic" overshoot models, and that the lower part of the convection zone is likely substantially subadiabatic. Overshoot has also been studied in detail using numerical simulations, with local regions (Brummell et al. 2002; Rogers and Glatzmaier 2005) and global 3-D domains (Brun et al. 2011).

4. Advances in modeling the Sun's internal dynamics

Having discussed the observational findings from helioseismology about the Sun's internal dynamics, we consider now what is understood from theory and numerical simulations.

4.1. Solar convection and mean flows

Recent insights into the nature of global-scale solar convection and mean flows have centered around two simple but powerful concepts. The first is thermal wind balance, which expresses the force balance in the meridional plane between the inertia of the differential rotation (Coriolis/centrifugal terms) and thermal gradients in latitude (baroclinic term):

$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta} \quad . \tag{3}$$

Here we use a mixture of spherical polar coordinates (r, θ, ϕ) and cylindrical coordinates (λ, ϕ, z), with S denoting the specific entropy. Angular brackets denote averages over longitude and time while g and C_P are the gravitational acceleration and the specific heat at constant pressure. Equation (3) holds if the meridional components of the convective Reynolds stress, the Lorentz force, and the viscous diffusion are small relative to the Coriolis and baroclinic terms. This is supported by a diverse range of theoretical and numerical modeling efforts and points to the central role of baroclinicity in accounting for the conical nature of the Ω isosurfaces inferred from helioseismic rotational inversions, as seen in Fig. 1 (Kitchatinov and Rüdiger 1995; Elliott et al. 2000; Robinson and Chan 2001; Rempel 2005b; Miesch et al. 2006; Balbus et al. 2009).

The second concept is that of gyroscopic pumping, which can be expressed by the following balance in the zonal component of the momentum equation:

$$\langle \rho \mathbf{v}_m \rangle \cdot \boldsymbol{\nabla} \mathcal{L} = \mathcal{F} \tag{4}$$

where $\mathbf{v}_m$ denotes the meridional components of the velocity, and $\mathcal{L} = \lambda^2 \Omega$ is the specific angular momentum. The right-hand side of eq. (4) is expressed as a generalized torque $\mathcal{F}$ but it can be loosely regarded as the negative divergence of the convective Reynolds stress. Lorentz forces and

viscous torques may also contribute to $\mathcal{F}$ but they are unlikely to be the central mechanism for establishing the solar differential rotation. For a derivation of eq. (4) and a thorough discussion of its implications, see Miesch and Hindman (2011).

Together these two concepts, reflected by the dynamical balances in equations (3) and (4), provide a theoretical foundation for interpreting helioseismic inversions and numerical models. For example, by making the additional ansatz that Ω and $\langle S \rangle$ isosurfaces coincide, Balbus and colleagues have shown that solutions to equation (3) coincide remarkably well with helioseismic rotational inversions. However, there are two caveats to this result. First, it does not explain why the solar equator rotates faster than the poles; adding an arbitrary cylindrical angular velocity component $\Omega'(\lambda)$ to Ω leaves equation (3) unchanged (geostrophic degeneracy) so a given $\langle S \rangle$ profile is consistent with an infinite number of Ω profiles, some solar-like (fast equator, slow poles), some anti-solar (slow equator, fast poles). Second, a compelling theoretical justification of why Ω and $\langle S \rangle$ surfaces should coincide remains an outstanding challenge (though see Balbus and Schaan 2012, for one perspective).

A puzzle that has received much attention recently is that of the subsurface structure of the meridional circulation. It has been realized since the pioneering work of Gilman (1977) that spherical convection simulations exhibit two rotation regimes, delineated by the Rossby number $R_o = V_c/(2\Omega L_c)$ where V_c and L_c are characteristic velocity and length scales for the convection. As R_o is increased across values of order unity, the differential rotation undergoes a transition from being solar-like to anti-solar, as illustrated in Fig. 8. More recent modeling efforts are clarifying this transition and assessing its implications for the meridional circulation (Käpylä et al. 2011, 2014; Gastine et al. 2013, 2014; Guerrero et al. 2013b; Hotta et al. 2014a,b; Featherstone and Miesch 2014). The fast-rotating regime generally exhibits multiple-cell profiles while the slow-rotating regime exhibits circulation profiles dominated by a single cell per hemisphere (Fig. 8). This transition can be understood in terms of a shift in the nature of the convective Reynolds stress $\mathcal{F}$ as discussed above in connection with eq. (4). The Sun is likely near the transition so it is unclear which meridional flow regime it may be in (Featherstone and Miesch 2014).

Gyroscopic pumping relies on the zonal component of the convective Reynolds stress, which induces meridional circulation by means of the Coriolis force. The meridional components of the Reynolds stress and Lorentz force can also establish meridional circulation by breaking the thermal wind balance (TWB; eq. 3). This occurs in many mean-field models that represent meridional Reynolds stresses as a turbulent diffusion and solve for the steady flow profiles (Kitchatinov 2012; Dikpati 2014)[1]. Departures from TWB occur particularly in the boundary layers, which can exert a disproportionate influence on the global meridional circulation profile. In particular, the shallow equatorward return flow inferred from recent helioseismic inversions and photospheric feature tracking (section 3.2) may be a boundary layer phenomenon, reflecting the penetration depth of

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[1]Though see Rempel (2005b) for an example of a time-dependent mean field model in which gyroscopic pumping is the dominant meridional flow driver.

surface-driven convective plumes (Featherstone and Miesch 2014).

Other current puzzles that are actively being investigated include the origin of the thermal gradients necessary for TWB (eq. 3) and the nature of the near-surface shear layer (NSSL). Proposed mechanisms for the former include the influence of rotation on convective heat transport (Kitchatinov and Rüdiger 1995; Brun and Toomre 2002; Käpylä et al. 2011), the influence of rotational shear on convective heat transport (Balbus et al. 2009; Balbus and Schaan 2012), and thermal coupling to the subadiabatic tachocline (Rempel 2005b; Miesch et al. 2006). For recent perspectives on the latter puzzle (the NSSL), see Miesch and Hindman (2011), Gastine et al. (2013), Guerrero et al. (2013b), and Hotta et al. (2014b).

4.2.   Magnetoconvection and surface magnetism

Magnetoconvective processes, i.e., the interaction between magnetic field and convective flows, play a central role in the generation, intensification, transport, and dissipation of magnetic flux in the convection zone and in the lower atmosphere of the Sun. Significant progress in our understanding of these processes has been brought about during the last decade by the combination of high-resolution observations and sophisticated numerical simulations (for recent reviews of various aspects of solar magnetoconvection, see Miesch 2005; Stein 2012; Weiss 2012; Schüssler 2013). Prominent examples are the formation of intergranular magnetic flux concentrations (Bercik et al. 1998; Vögler et al. 2005; Stein and Nordlund 2006; Schaffenberger et al. 2006) and magnetized vortices (Vögler 2004; Shelyag et al. 2011; Moll et al. 2012; Wedemeyer-Böhm et al. 2012; Kitiashvili et al. 2012; Shelyag et al. 2013), the near-surface structure and dynamics of sunspot umbrae and penumbrae (Schüssler and Vögler 2006; Heinemann et al. 2007; Rempel et al. 2009a,b; Rempel 2011a,b, 2012), the emergence of active regions (Cheung et al. 2007; Martínez-Sykora et al. 2008; Cheung et al. 2010), as well as small-scale dynamo action in the deep convection zone (Brun et al. 2004) and in its near-surface layers (Vögler and Schüssler 2007; Pietarila Graham et al. 2010).

Recent developments concern magnetoconvection simulations in computational boxes of down to 50 Mm depth, so that the effect of flows at supergranular scales can be studied. These flow patterns imprint their signature on the distribution of magnetic flux at the visible surface, leading to mesogranular and supergranular network patterns. If a sufficient amount of vertical background flux is present, bigger flux concentrations resembling observations of dark pores are formed by flux expulsion and suppression of convective energy transport. An important requirement for this kind of simulations is a sufficiently big aspect ratio of the computational box (ratio of horizontal size to depth): since the largest flow structures typically have a horizontal extension of the order of the box depth, the horizontal size of the box should be at least 3-4 times its depth, so that a sufficiently large number of cells is present, thus minimizing the effect of the (typically periodic) side boundaries. Results based on simulations with aspect ratios of unity (e.g., Kitiashvili et al. 2010) should therefore be considered with some caution.

Fig. 9 shows magnetic flux concentrations and pore-like structures forming in simulations in a box of $24\,\mathrm{Mm} \times 24\,\mathrm{Mm}$ horizontal size and $6\,\mathrm{Mm}$ depth. The simulations differ by the amount of imposed vertical background flux: in the case with a horizontally averaged vertical field of $\langle B_z \rangle = 100\,\mathrm{G}$ (upper panels), a patchy network on mesogranular scales and a few micropores formed; for $\langle B_z \rangle = 400\,\mathrm{G}$ (lower panels), the network is more pronounced and a number of dark pores of the size of a few granules are present. Note that $\langle B_z \rangle = 400\,\mathrm{G}$ is already at the high end for solar plage areas; consequently we do expect even bigger structures (sunspots) to form spontaneously from existing background flux. This is consistent with the well-known observational fact that sunspots invariably form in the course of flux emergence and never from pre-existing flux in a mature plage region.

Figure 10 shows a comparison of flows and magnetic field patterns for various depths. The corresponding magnetoconvection simulation ($\langle B_z \rangle = 100\,\mathrm{G}$) was carried out by M.C.M. Cheung in a computational box with $49.2\,\mathrm{Mm} \times 49.2\,\mathrm{Mm}$ horizontal size and reaching down to about $14\,\mathrm{Mm}$ below the optical surface. The various vertical flow patterns present at the different depths up to about supergranular scale are reflected in the multi-cellular distribution of vertical magnetic flux at the surface, owing to flux expulsion by the corresponding horizontal flows patterns. This offers the possibility to compare the simulation results with actual observations. Since the average properties of the simulated convection (e.g., depth profiles of horizontally averaged thermodynamic quantities, horizontal scales and velocities) are in good agreement with mixing-length models, this would also provide a consistency check for such models and shed light on the question whether solar convection might actually work in a completely different, essentially unmixed regime with cool, narrow downdrafts traversing the whole convection zone and driving a very slow, broad upflow (Spruit 1997). Such a regime appears to be favored by recent indications from helioseismology (Fig. 7). Such 'slow', unmixed solar convection would have severe consequences for models of the solar dynamo and for the generation of differential rotation and meridional flow (Miesch et al. 2012, see also the discussion in Section 3.3 above). Existing numerical simulations invariably show well mixed convection with velocities consistent with mixing-length models. This is the case for anelastic simulations of convection in the deep convection zone (e.g., Miesch 2005) as well as for compressible simulations in the upper layers of the convection zone, which include the driving by radiative losses in the photosphere (e.g., Trampedach and Stein 2011). If solar convection actually would work in an essentially unmixed regime, then which are the critical Reynolds (and perhaps Prandtl) numbers for the transition away from the mixing-length regime? When can we expect to see numerical simulations of this transition, if it exists?

5.   Future directions

Reliable seismic inferences on the interior structure of prominent solar phenomena such as sunspots, supergranulation, convection and meridional circulation would have significant consequences not only for our understanding of the way the Sun operates but also for Sun-like stars.

Understanding magneto-convection and the emergence and sustenance of large-scale field in the Sun would provide important constraints for dynamo theory. Thus the seismic science of the Sun can play a critical role in advancing astrophysics as a whole.

While global helioseismology, the study of large-scale axisymmetric structure in the Sun, has met with great success, local helioseismology, which deals with spatially localized features such as sunspots and supergranules and convection in the interior, has a long road ahead. Multi-variable inverse problems are inevitable in local helioseismology, where seismic measurements, obtained at the photosphere, must be related to a large number of parameters, e.g., the sound speed, flows and magnetic fields in the interior. In particular, the full accounting of systematical effects, finite wavelengths and inversion non-linearity, which can significantly influence and bias results, would greatly improve their trustworthiness and accuracy of local helioseismic inferences. The future of a theoretically sound seismology holds the promise of revealing important insights such as the distribution of large-scale Reynolds stresses, the drivers of differential rotation and meridional circulation, and ultimately, possibly the source of global magnetism in the Sun itself.

Meanwhile, comprehensive simulations of radiative (magneto-)convection in the near-surface layers of the Sun have achieved an impressive realism. They compare well with observational results (e.g., Nordlund et al. 2009; Stein 2012) as well as between different codes (Beeck et al. 2012).

In the deeper layers, the average properties of such simulations are similar to the results of mixing-length models and we can expect them to make contact with the anelastic simulations soon as well. Does that mean that the solar convection zone is in a well-mixed regime in the sense that the low-entropy downflows are well mixed into the high-entropy upflows? Or are simulations in the wrong regime owing to their much too small Reynolds numbers? The latter view seems to be supported by some results of local helioseismology, but these need to be confirmed before definite conclusions can be drawn. If they turn out to be correct, how can we then reconcile the fact that the near-surface simulations (which have much too low Reynolds numbers as well) are in such an excellent agreement with observations, so that even significant corrections of element abundances of fundamental astrophysical importance can be derived from them (Asplund et al. 2009).

We dedicate this paper to our colleague and friend Dr. Irene González Hernández (1969-2014), a pioneer of ring-diagram analysis and far-side imaging, who passed away on 14 February 2014.

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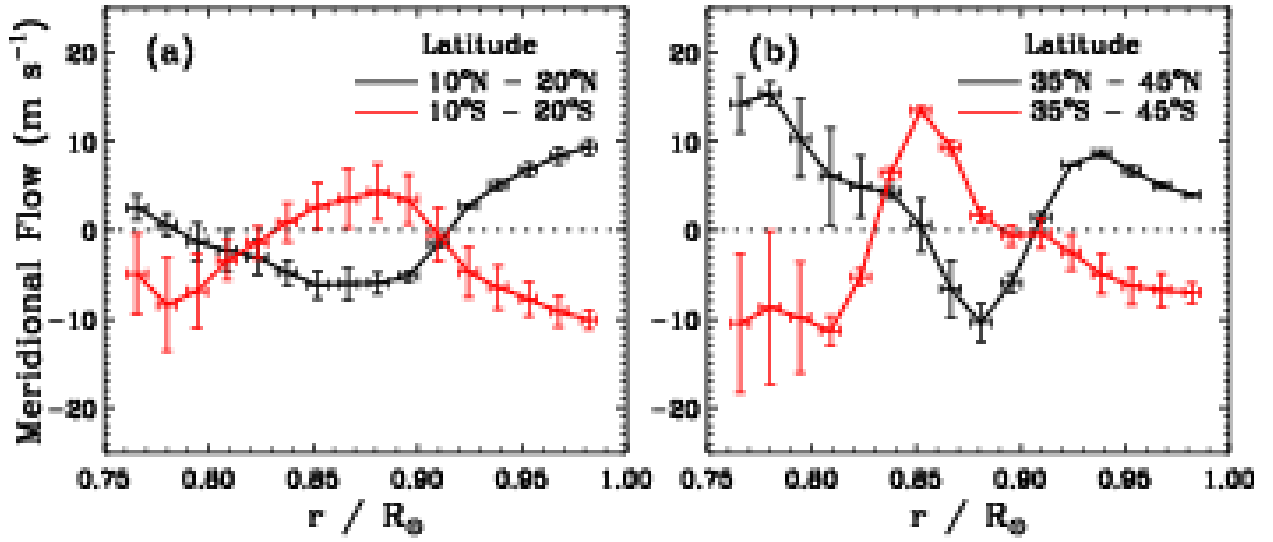
Fig. 3.— Meridional flow profile as a function of radius at selected latitudes in the northern (black) and southern (red) hemispheres, obtained from time-distance helioseismology using HMI observations (Zhao et al. 2013). In the near-surface region the flows are poleward in both hemispheres, but the results indicate that there is a counter-cell beneath radius $0.90R_\odot$. There may be yet another cell beneath about $0.82R_\odot$, but the errors on the inferred flow are larger at such depths. (Figure courtesy of Junwei Zhao)

Fig. 4.— Surface peak velocities of the horizontal meridional flow component of degree 2 as a function of ν/L (Woodard et al. 2013). (ν/L relates directly to the radial location r_t of the lower turning point of the mode: e.g., for $\nu/L = 10\mu Hz$, $r_t \simeq 0.98R_\odot$, while for $\nu/L = 100\mu Hz$, $r_t \simeq 0.6R_\odot$.) The analysis is based on fitting a model function to the cross-spectra between p-modes and is based on 500 days of HMI data.
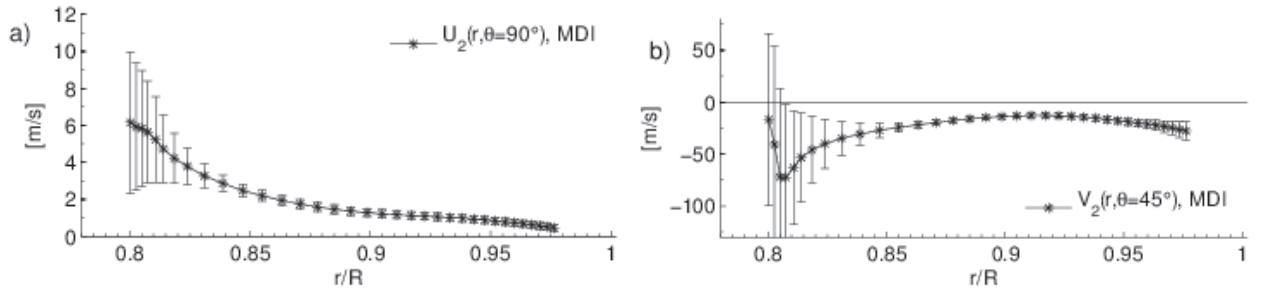

Fig. 5.— The radial and horizontal amplitudes of the meridional flow component with harmonic degree 2 as a function of radius r/R at colatitude $\theta = 90°$ (equator; left) and $\theta = 45°$ (mid-latitude; right), respectively. The flow was measured by evaluating cross-spectral amplitude ratios obtained from MDI data covering the period 2004–2010. Positive values of U_2 indicate an upward radial flow, negative values of V_2 refer to a poleward flow (Schad et al. 2012).
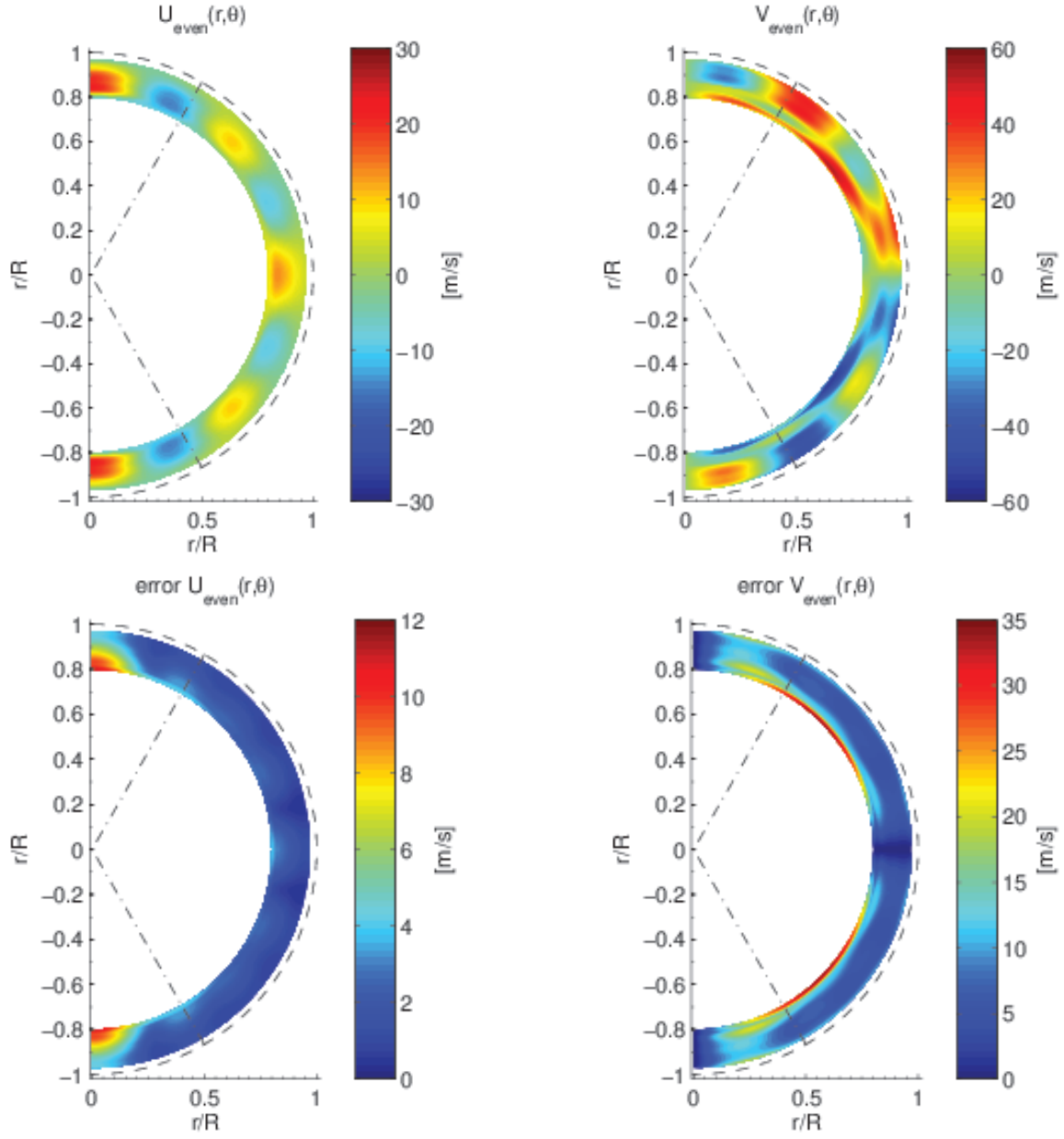
Fig. 6.— Cross-sections through the meridional flow composed of the harmonic degrees 2, 4, 6, 8 (top) as a function of fractional radius and latitude based on evaluating cross-spectral amplitude ratios obtained from MDI data covering the period 2004–2010 (Schad et al. 2013). The dashed-dotted lines mark the latitudes of 60°. The radial flow is displayed on the left where positive (negative) values correspond to outward (inward) directed flows. The horizontal flow is displayed on the right; positive (negative) values correspond to northward (southward) directed flows. The 1σ standard error of the composite flow is given in the lower panels.
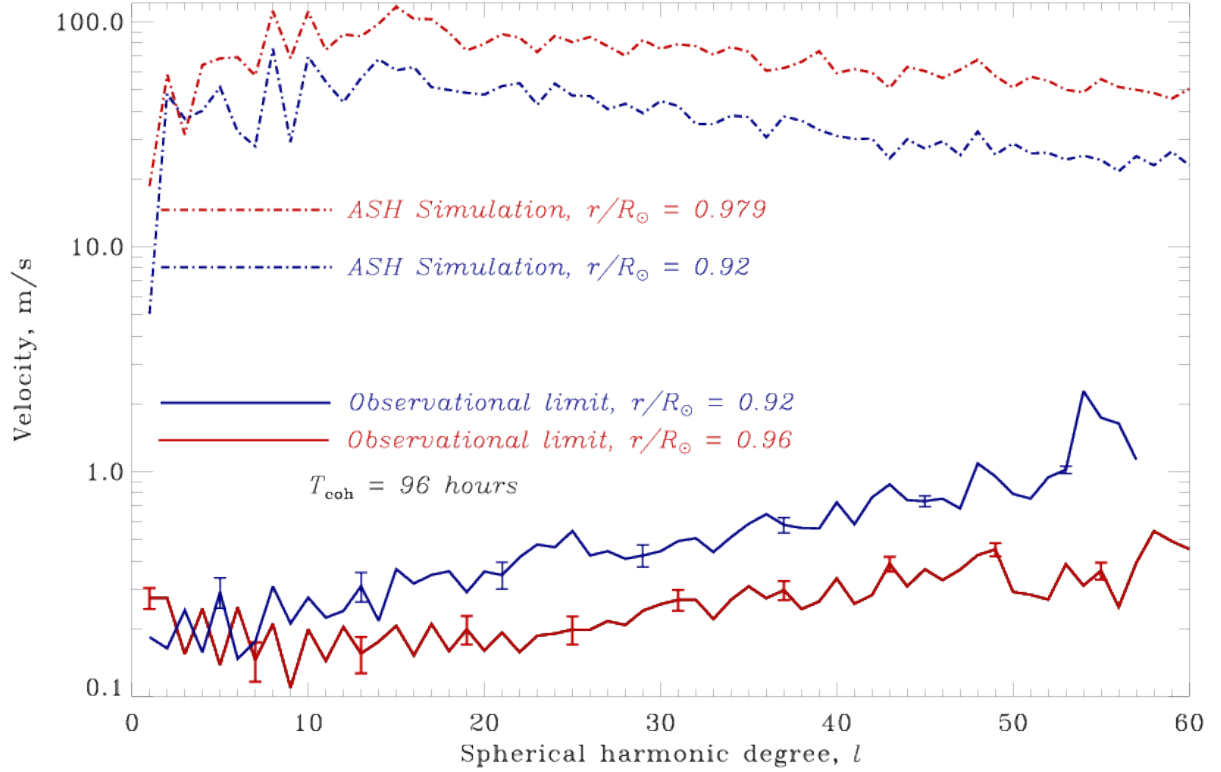
Fig. 7.— Seismic constraints obtained by Hanasoge et al. (2012) using data from HMI (Schou et al. 2012). Assuming a convective coherence time of 96 hours, Hanasoge et al. (2012) obtained upper bounds on the observed convective spectrum. Shown in comparison is the Anelastic Spherical Harmonic (ASH) convective spectrum. These differences suggest that convection in the Sun may be operating in a strongly non-MLT regime.
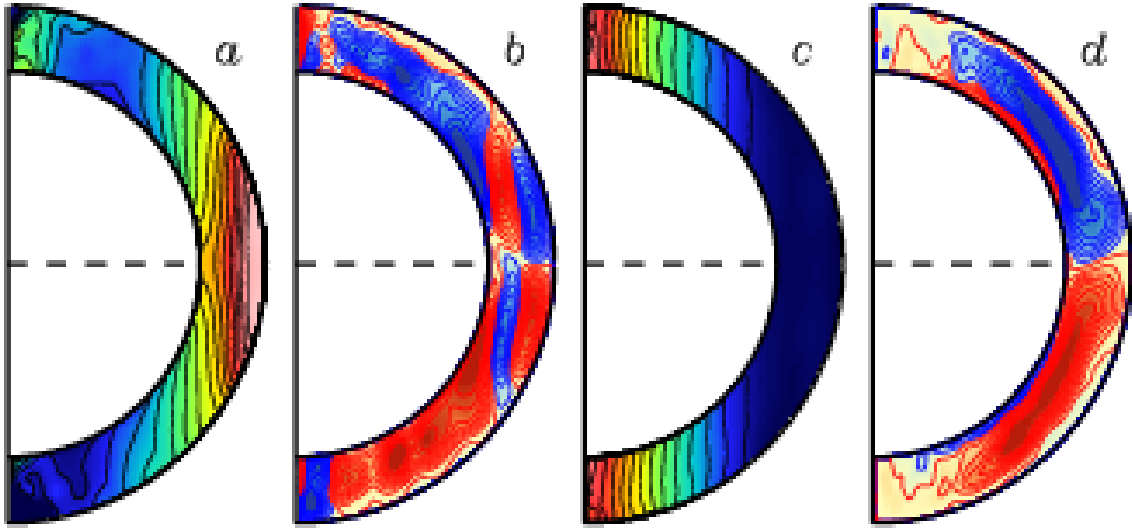
Fig. 8.— Mean flow regimes in spherical convection, from Featherstone and Miesch (2014). Shown are the differential rotation (a, c) and meridional circulation profiles (b,d)
in two simulations of global solar convection. Pink/yellow and blue/black tones denote faster and slower rotation in frames a,c while red and blue denote clockwise and counter-clockwise circulation in b, d. On the left (a,b) is the rapidly-rotating regime characterized by a solar-like Ω profile and multi-celled meridional circulation. On the right (c,d) is the slowly-rotating regime characterized by an anti-solar differential rotation and and a single dominant circulation cell per hemisphere. The Sun may be near the transition.
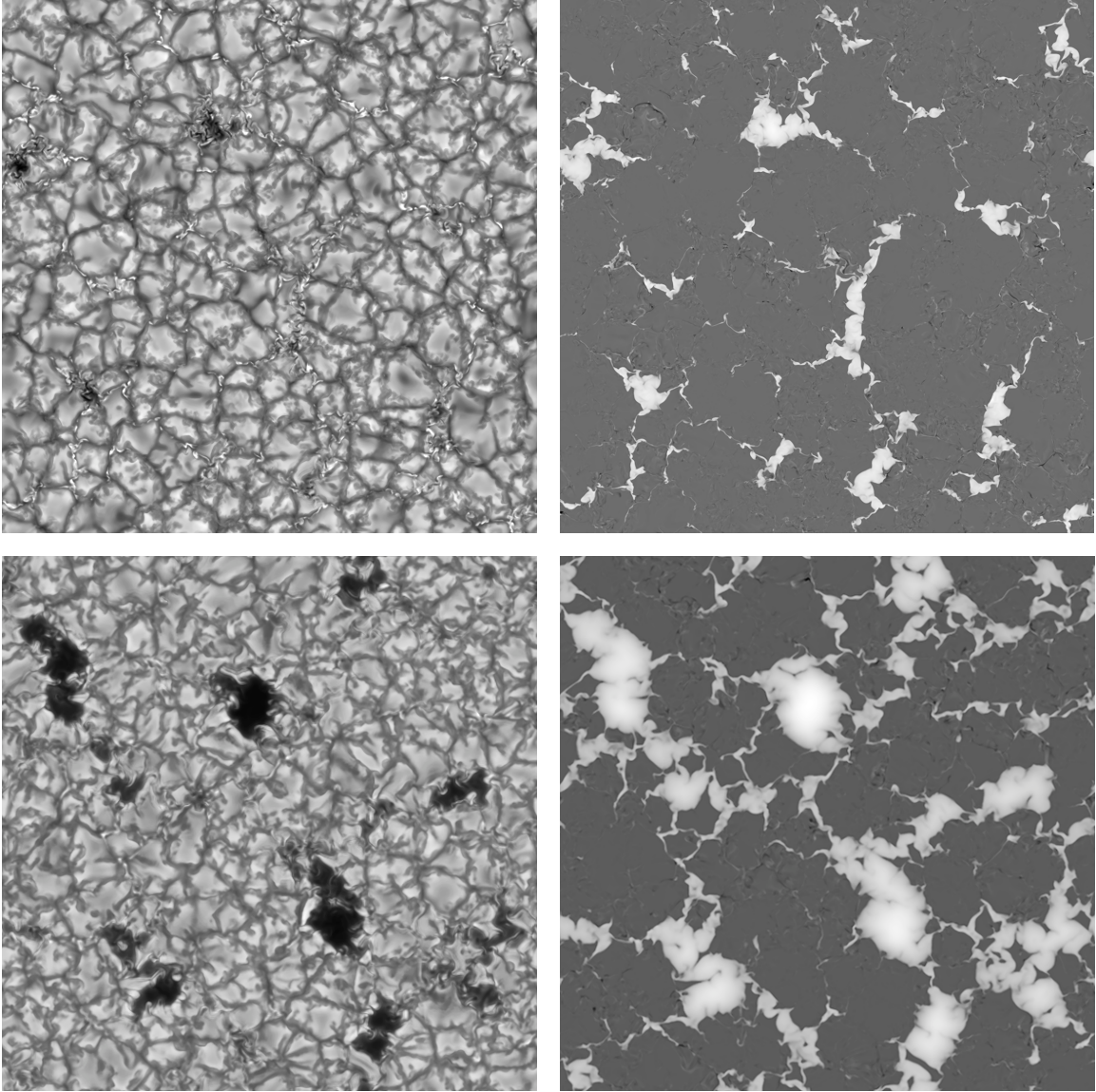
Fig. 9.— Maps of bolometric brightness (left panels) and vertical magnetic field at the optical surface (right panels; dark grey represents very weak field) from magnetoconvection simulations with average vertical background fields of 100 G (upper row) and 400 G (lower row). The horizontal extension of the computational box was 24 Mm × 24 Mm and the depth 6 Mm. The bigger amount of magnetic flux in the case with 400 G background field leads to the formation of proper pores while only micropores are present in the case with 100 G. The latter case has a twice higher horizontal grid resolution (20.8 km), which leads to a better representation of the small-scale bright flux concentrations in the intergranular lanes.
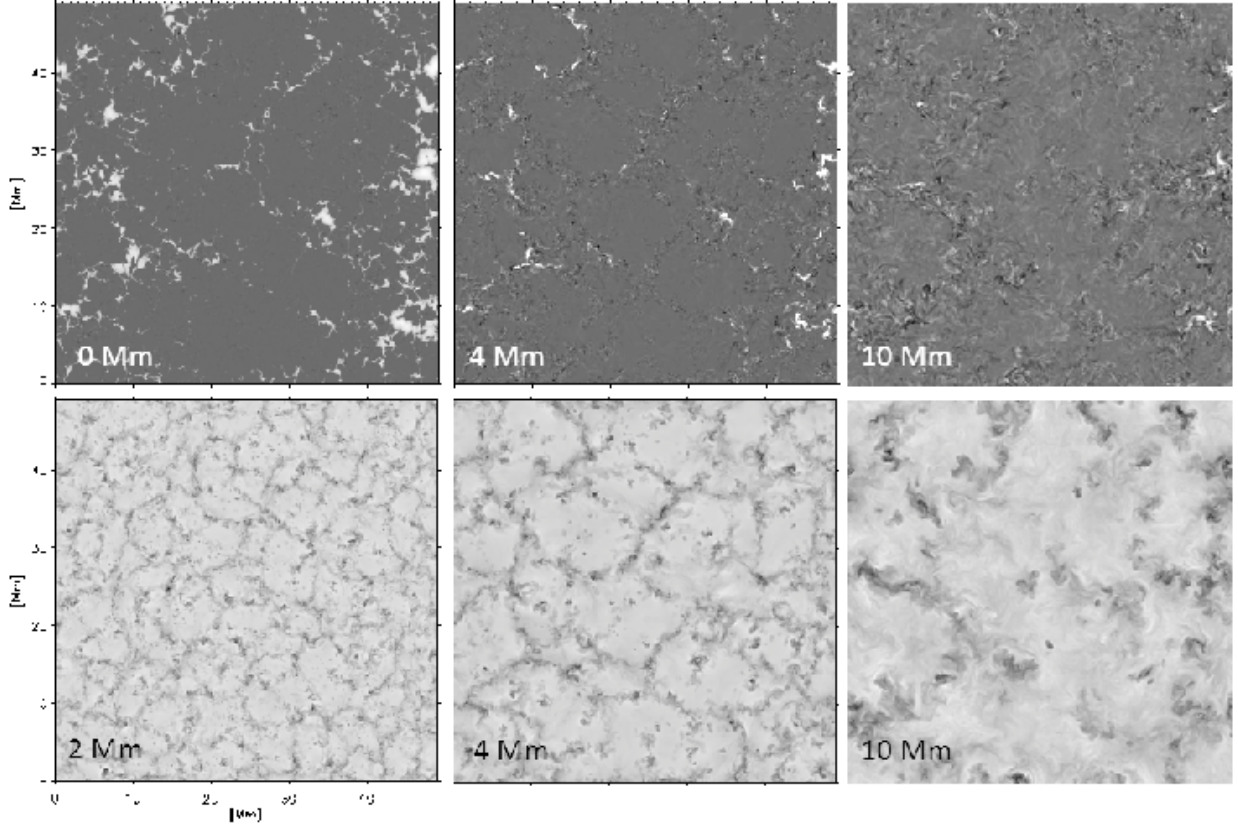
Fig. 10.— Maps of the vertical magnetic field (upper panels; dark grey represents very weak field) and vertical flow velocity (lower panels; darker shades represent downflows, brighter shades upflows) at various depths below the optical surface from a magnetoconvection simulation with average vertical background fields of 100 G (courtesy of M.C.M. Cheung). The horizontal extension of the computational box was 49.2 Mm × 49.2 Mm and the depth 15.4 Mm. The spatial patterns of the observable surface field (upper left panel) reflect the horizontal scales covered by the flow patterns in the depth range of the simulation.